REGULAR TOPOLOGIES APPLICATION ON MULTILEVEL NETWORKS

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ABSTRACT
Multi-level networks have been a good solution in large scale network scenarios. The implementation of a network in different levels or sub-layers, improves the performance and reduces the investment compared to flat topologies. This paper introduces a method to characterize important parameters on multi-level networks such as diameter, average distance and gateway location to be able to optimize the global network topology. The study focuses on the lower level of the network formed by subnetworks with regular structures, and the method is illustrated with two of the simplest regular topologies such as Single Ring and Double Ring. The conclusions achieved will ease and improve the network planning of large scale networks.

KEY WORDS
Multilevel Networks, Subnetwork, Single Ring, Double Ring, Diameter, Average Distance

1 Introduction

The network interconnection planning has to deal with the main properties: *node degree* (links at the nodes) and *diameter* (maximum path distance for any pair of nodes). The ideal network would have a low node degree that optimizes the economical investment on the network and low diameter that optimizes the performance of the network. The problem is that these two properties are contradictory [1]. Low degree networks usually involve a high diameter and vice versa. The problem becomes critical with large scale networks where the performance or the investment will not fulfil the requirements due to the size of the network. In scenarios with large number of nodes implemented as a flat topology the two options are to build a high degree structure which involves a high investment but short distances or the opposite [1].

Multi-level networks have become an option for large scale interconnection network schemes obtaining better properties, such as network diameter and average distance, than mono-level networks. These performance properties improve the Structural Quality of Service (SQoS), a number of metrics and properties related with the logical structure of the network [2].

The possibility of using a mechanized and simple method to decide the structure for a multilevel network would ease the planning tasks. A new network planning method is proposed based on mathematical properties of regular topologies for multilevel networks. This method will allow to define and characterize the performance of a network based on known regular topological information about their subnetworks. The key aspect of this depiction is to perform the characterization of using regular topologies without the need of additional path calculations, which is unavoidable in the case of irregular topologies.

The study of regular topologies in mono-layer networks identifies relational patterns on the topology parameters, such as diameters and average distances, as a function of the number of nodes and independent paths. Extrapolating these ideas, it is possible to define similar patterns for multilevel environments.

This paper will only focus on the lowest level subnetworks and in future the higher lever will be studied. The idea is to treat each subnetwork as independent and all the subnetworks are connected to a *Black box*, representing the higher levels of the network. This idea of dividing the network into independent groups is not new. It has been successfully used to plan a two level network using heuristic algorithms. [3].

Two of the simplest regular topologies are chosen to present the bases for this network planning proposal (Single and Double Ring). These two structures have been extensively analyzed on mono-level environments, but surprisingly, they have not been characterized for a multilevel purpose. The study of these two topologies is not enough for efficiently planning a multilevel network, many other regular topologies should be characterize in the same way. Therefore, this work tries to illustrate the process, methodology and feasibility of this new method, but to be applied on real scenarios there is much work that has to be done.

The structure of the rest of the document is as follows Section 2 introduces the previous and related work to this topic. Section 3 explains the parameters used to characterize the network topologies. Section 4 explains the procedure of obtaining the mathematical properties and equations. Section 5 introduces formulas obtained for the two analyzed topologies. Finally, in Section 7 the conclusions are extracted from this paper.

2 Related work

Multilevel networks are already being used, especially for distribution networks [4] and [5]. The multilevel connections have been treated from different points of view:

In [6]-[7] the goal was to obtain a solution using algorithms, such as paths calculations, to interconnect a set of nodes optimizing the diameter and arcs connecting the nodes. This optimization can be done only for small networks due to the exponential growth of the number of paths calculated. In [8] the performance of routing at different link configurations are tested and in other cases a regular topology is studied to find the way of dividing it in multi-levels such as in [9] and [10]. However, these studies do not treat the combination of different types of topologies.

3 Parameters

This section introduces the important concepts of the analyzed parameters to characterize the network topologies:

Diameter ($D_{T,N,L,P}$): This value corresponds to the maximum number of hops needed to leave the subnetwork to the higher or lower level. This value is calculated in function of:

- *T*: Topology (SR or DR).

- *N*: Number of nodes at the subnetwork.

- *L*: Number nodes or gateways linked to the higher level (two or three).

- *P*: Path number (1 corresponds to the shortest path, 2 to the second independent shortest path).

Gateways are the nodes linking levels named as G_j being $0 < j \leq L$, each subnetwork will have *L* gateways. In this work, the number of gateways is limited to three to be able to contract a more vertical structure, the more gateways a subnetwork has, more nodes will form the higher level. For logical representation each gateway is linked to another node belonging to the higher level, but in reality, these gateways nodes would probably be located in the same building and physically belonging to both levels. The reason for considering two separate nodes is that logically the process of level changing even at the same node can be considered as one hop just by itself.

Let $d_i(P)$ be the shortest distance ($P = 1$) or the second shortest distance considering no links nor nodes in common with the first path ($P = 2$) from any node *i* to the closest gateway G_j. Then the diameter value is given by Equation (1).

$$D_{T,N,L,P} = max(d_i(P)) \tag{1}$$

Average distance ($A_{T,N,L,P}$): This value corresponds to the average number of hops needed to leave the subnetwork to the higher level or vice versa in function of the same parameters as the diameter.

The value of the average distance, considering the same rules as the diameter, for $P = 1$ and $P = 2$ is given by Formula (2).

$$A_{T,N,L,P} = \frac{1}{N}\sum_{i=1}^{N} d_i(p) \tag{2}$$

4 Methodology

This section treats the process of obtaining the equations of the different cases. The idea is to calculate the distances from all the nodes to the gateways (nodes linked to the higher level). Due to the topological properties of regular structures it is possible to calculate the distances with no path calculation at all. The number of independent or disjoint paths treated in this paper are just two, but in the future a third or fourth path analysis can be included (for the topologies that allow them, the number of independent paths is given by the node degree of the topology).

To find the best position of the gateways at the network, an implemented algorithm performs a sweep of the possible combination of the nodes selected. The mechanism of the algorithm is to select one of the nodes (it can be any node due to the regularity of the topologies studied) that can be named as G_1. Then starting to sweep for all the possible values of the rest of the gateways the parameters are calculated (diameters and average distances) and the best options are selected as G_2 and G_3 (in case of three links to the higher level). The result obtained gives the relative positions of G_2 and G_3 from G_1, and these relative positions can be expressed as a function of the number of nodes *N*, explained in depth in Section 5. G_1 can be any of the nodes of the network, therefore there will be *N* optimal configurations for the position of the gateways.

The best configurations change depending on the considered parameters, in this case, and in order, they are the minimum values of the following criteria:

1) Diameter of the first path ($D_{T,N,L,P=1}$)

2) Diameter of the second path ($D_{T,N,L,P=2}$)

3) Average distance of the first path ($A_{T,N,L,P=1}$)

4) Average distance of the second path ($A_{T,N,L,P=2}$)

The reason for giving priority to the diameter values is due to their role when guaranteeing a certain level of performance for the network. These values can be considered as the worst case possible. Therefore, there is a defined limit for communications between any pair of nodes (in this case from a node to a corresponding gateway).

The rest of the simulation consists of the increment of the number of nodes *N* which will give deterministic series in function of *N* for the four parameters studied. Based on these series, mathematical formulas are defined to characterize each of the topologies in Section 5.

5 Mathematical Approach

This section treats the equations and parameters found that characterize the subnetworks and linking the higher/lower level(s) of the network. As an introduction, the following paragraphs introduce the properties and formats of the formulas obtained for a better understanding.

Diameter:

Usually, the diameter follows a stair distribution with *N*. The difference between $D_{T,N,L,P}$ and $D_{T,N-1,L,P}$ is always 0 or 1 hop, which defines the increment of the stair step on 1 hop respect to the previous one. The same value of $D_{T,N,L,P}$ is related to a set of consecutive values of *N* (*#SET*). The value of *#SET* is constant in each of the cases (the same for the same values of *T*, *L* and *P*). Another value called offset, O_f, is related to the starting value of N of each of the sets with the same diameter. The diameter formulation is expressed in a general formula as Equation (3).

$$D_{T,N,L,P} = \left\lceil \frac{N + O_f}{\#SET} \right\rceil \quad (3)$$

Average Distance:

The equations obtained for the average distances usually follow a pattern which can be defined as a sum that corresponds to Equation (4) where *IC* is the initial condition of the series and *S* is the increment or step in the series.

$$A_{T,N,L,P} = IC + \sum S(STEP) + E_{T,N,L,P} \quad (4)$$

Due to the asymmetric relation between *N* (number of nodes) and *L* (links to the higher level) the series has anomalies for some of those *N* values. The variable $E_{T,N,L,P}$ is introduced to correct this problem and will have the format illustrated by Equation (5) where *REP* is the cycle of the anomaly which is constant for the same values of *T, L* and *P*; N_{err} is any value of N where the anomaly occurs and E is the value of the anomaly.

$$E_{T,N,L,P} = \quad E * (\left\lceil \tfrac{N+REP-N_{err}}{REP} \right\rceil - \left\lceil \tfrac{N+REP-N_{err}-1}{REP} \right\rceil) \quad (5)$$

Analyzing Equation (5), $E_{T,N,L,P} = 0$ for all *N* values different than N_{err} and at the exact values of N_{err} $E_{T,N,L,P} = E$, which is required to have a modification on the series.

Gateway position:

This value represents the relative position of the nodes linked to the higher level to obtain the best results possible concerning the two previous values. The relative position is only needed to be calculated from a fixed value of G_1 due to the regularity of the structure and then the result can be applied to any possible value of G_1. The notation is defined by Equation (6):

$$\begin{array}{l} G_1 \\ G_2 = (G_1 + X)\mathit{mod(N)} \\ G_3 = (G_1 + Y)mod(N) \end{array} \quad (6)$$

Being $1 \leq G_1 \leq N$ and *X* and *Y* positive integers, $X < Y$, considering mod(N) for any resulting position of G_2 or G_3. In most of the cases, there is more that one solution for the relative position of the gateways.

5.1 Single Ring (SR)

The Single Ring analysis gives simple equations very useful to identify each term explained at the introduction of this Section. The diameter value ($D_{SR}(N, L, P)$)is shown by Equation (7):

$$D_{SR}(N, L, P) = \left\lceil \frac{(N + L - P) * P}{2L} \right\rceil \quad (7)$$

There are some interesting conclusions extracted from Equation (7). The value $\#SET$ is given by $2L/P$ and O_f corresponds to the term $L - P$.

The value from Table 1 for the SR helps to define a general equation for the average distances. The average distance ($D_{SR}(N, L, P)$)is related with $D_{SR}(N, L, P = 1)$ and it corresponds to Equation (8):

$$A_{SR}(N, L, P) = \frac{(2 * P - 1)}{N} * \sum_{i=3}^{N} \underbrace{\left\lceil \frac{i + L - 1}{2L} \right\rceil}_{D_{SR}(i,L,P=1)} + E_{SR}(N, L, P) \quad (8)$$

The term $E_{SR}(N, L, P)$ corrects the variation of the series due to some asymmetries between the number of nodes and *L* and its value can be found at Table 2. The conclusion of Equation (8) is that the value of S (the step of the series) corresponds to $(2 * P - 1) * \left\lceil \frac{i+L-1}{2L} \right\rceil$. As *N* increases, the step increases as well. The term $2 * P - 1$ indicates that the average distance of the second path ($A_{SR}(N, L, P = 2)$) is three times the average distance of the first path ($A_{SR}(N, L, P = 1)$) in most of the cases (when $E_{SR}(N, L, P) = 0$), see Equation (9)). There is no term IC for any of the situations.

$$A_{SR}(N, L, 2) = 3 * A_{SR}(N, L, 1) + E_{SR}(N, L, 2) \quad (9)$$

The term $E_{SR}(N, L, P)$ of Equations (8) and (9) can be summarized by Equation (10).

$$E_{SR}(N, L, P) = \begin{cases} \frac{4-L+2*(N(mod\ 2))}{N} & \text{if condition (11)} \\ 0 & \text{rest} \end{cases} \quad (10)$$

$$\begin{array}{l} \left\{ \frac{N+X}{L} \right\} = 0, \ \ \frac{N+X}{L}(mod\ \ 2) = 1 \ \ \& \ \ P = 2 \\ -(L - 2) \leq X \leq (L - 2) \end{array} \quad (11)$$

The gateways positions at Tables 4 and 5 represent the relative position of the nodes linked to the higher level in the subnetwork. For most values of *N*, there is more than one solution for these relative positions. To identify these solutions and the corresponding *N* for each one, *N* can be defined as a function of a variable $k \in \mathbb{N}$

The values of $V_{SR,1}$, $V_{SR,2}$ and $V_{SR,3}$ presented at Tables 4 and 5 are shown at Table 6. The values of $V_{SR,1}$ and $V_{SR,3}$ are independent but $V_{SR,2}$ depends on $V_{SR,1}$.

There is a direct relation between the relative position of the gateways and the number of optimal configurations with the value of *L*. Making the proper substitutions in the equations the number of different optimal solutions for the relative position of the gateways can be identified, the total number of optimal configuration is *N* times these values since G_1 can be any node of the SR. See Table 8.

5.2 Double Ring (DR)

The study of the Double Ring gives some simple equations to be able to define a general equation for the diameter values. See Equation (12).

$$D_{DR}(N, L, P) = \left\lceil \frac{(N - O_{fDR}) * P}{4L} \right\rceil + 1 + E_{DRDiam} \quad (12)$$

The term $E_{DR(Diam)}$ is an anomaly on the series for the value of the diameter $D_{DR}(N, L = 3, P = 2)$. The anomaly in this case comes from the chosen positions of the gateways. As explained at Section 4 the first criterion to decide the optimal position of the gateways is to optimize the first path diameter ($D_{DR}(N, L = 3, P = 1)$). In this case the optimization costs an increment of the value of the second path diameter ($D_{DR}(N, L = 3, P = 2)$). If the criterion would be to optimize ($D_{DR}(N, L = 3, P = 2)$) then the anomaly would be at the $D_{DR}(N, L = 3, P = 1)$ equation. In this case $E_{DR(Diam)}$ corresponds to formula (13).

$$E_{DR(Diam)} = \begin{cases} 1 & \text{if } \{\frac{N+2}{6}\} = 0, P = 2 \text{ and } L = 3 \\ 0 & \text{rest} \end{cases} \tag{13}$$

The same conclusions can be extracted from Formula (12) as from Formula (7) in the case of the SR. In this case the term $P/4L$ gives the value of $\#SET$. The value of O_f is presented at Condition (14). Of is 2 when $L = 2$ and $P = 1$.

$$O_{fDR} = \begin{cases} 2 & \text{if } P = 1 \text{ and } L = 2 \\ 0 & \text{rest} \end{cases} \tag{14}$$

Unfortunately for the average distances ($A_{DR}(N, L, P)$) it was not possible to find a general equation based on the results presented at Table 1. In any case, looking at each equation individually, the terms S and IC are easily identified.

Tables 3 and 4 show the optimal relative position of the gateways at the DR topology. The value of N is defined as a function of a variable $k \in \mathbb{N}$, as at the SR case, to identify the different solutions. Table 6 exposes the values of the variable used at Table 4, $V_{DR,1}$, $V_{DR,2}$ and $V_{DR,3}$, in this case as a function of $V_{DR,2}$ since $V_{DR,3}$ depends on it.

In the case of $L = 3$ the location of the gateways equations becomes complex. Therefore, the gateways possibilities are split in m options where $1 \leq m \leq Number\ of\ possibilities$ and they are treated separately from the positions when $L = 2$.

The division basically is following the scheme represented at Table 3, in function of m and the values $V_{DR,4}$, $V_{DR,5}$, $V_{DR,6}$, and $V_{DR,7}$ (Table 7) to represent the different solutions for the same configurations.

Making the proper substitutions on the equations on Table 3 with the values of Tables 6 and 7 the number of possible optimal solution for the relative position of the nodes is presented at Table 8.

SR	G_2
$N = 4 * k$	$G_1 + \frac{N}{2} + V_{SR,1}$
$N = 2 * k + 1$	$G_1 + \left[\frac{N+V_{SR,2}}{2}\right]$
$N = 2 * (2 * k + 1)$	$G_1 + \frac{N}{2}$
DR	G_2
$N = 8 * k$	$G_1 + \frac{3*N}{4}$
$N = 2 * (2 * k + 1)$	$G_1 + \left[\frac{N}{4}\right] + V_{DR,1}$
$N = 4 * (2 * k + 1)$	$G_1 + \frac{V_{DR,2}*N}{4} + V_{DR,3}$

Table 4. Gateways Relative Position When L=2

N	G_2	G_3
$6 * k$	$G_1 + \frac{N}{3} + V_{SR,1}$	$G_1 + \frac{2N}{3} + V_{SR,2}$
$3 * k + 1$ **&** $3 * k + 2$	$G_1 + \left[\frac{N+V_{SR,3}}{3}\right]$	$G_1 + \left[\frac{2N+V_{SR,3}}{3}\right]$
$3 * (2 * k + 1)$	$G_1 + \frac{N}{3}$	$G_1 + \frac{2N}{3}$

Table 5. Single Ring, Gateways Position When L=3

SR	$V_{SR,1} = -1$	$V_{SR,1} = 0$	$V_{SR,1} = 1$
$V_{SR,2}$	-1,0	-1,0,1	0,1
$V_{SR,3}$	$(0, L-1)$	$(0, L-1)$	$(0, L-1)$

DR	$V_{DR,2} = 1$	$V_{DR,2} = 3$
$V_{DR,3}$	0	-1,0,1
$V_{DR,1}$	$0, 1$	$0, 1$

Table 6. V values

6 Application

It is important to understand that the parameters equations must be defined only once. Then, the can be used for planning a real network. In this paper it has been already described how to define the formulas for multilevel purposes for two topologies. The important issue is the methodology and concept of obtaining the formulas for any regular topology, more important than the resulting formulas presented for the Ring or Double Ring which are just examples of the procedure.

6.1 Topology decision

The best way to explain the application of the proposed method is to set an scenario to test it. It should be always kept in mind that this example is a basic illustration of the application of the method, for the real feasibility of it, more topologies must be included on the analysis.

The test scenario consists on three municipalities, each of them is required to have a network by their own and at the same time they should be interconnected to each other. Therefore, each municipality is a subnetwork and they are interconnected by a higher level. The number of nodes required at each subnetwork are 12, 20 and 30. For reliability reasons two independent or disjoint paths must be provided between any pair of nodes of the complete network. It is assumed that the higher level (which is out of the scope of this study) can handle the requirements of two independent paths for any pair of nodes. The goal is to design the best topology for each subnetwork but always considering the fact that they should have similar logical properties to obtain the best possible performance.

Applying the formulas presented in Section 5, Table 9 shows the results for each of the subnetwork and each of the possible networks (SR and DR and $L = 2$ and $L = 3$) for two independent paths.

The decision of the topologies at this point is very simple. The subnetwork with 30 nodes is the most restrictive. Hence, the best topology possible is selected for this municipality since the goal is to achieve the best possible performance. Following the criteria described in Section 4

SR	L=2	L=3
P=1	$\frac{1}{N} * \sum_{i=3}^{N} \left[\frac{i+1}{4}\right]$	$\frac{1}{N} * \sum_{i=3}^{N} \left[\frac{i+2}{6}\right]$
P=2	$\frac{1}{N} * \sum_{i=3}^{N} 3 * \left[\frac{i+1}{4}\right] + E_{SR}(N,L,P)$	$\frac{1}{N} * \sum_{i=3}^{N} 3 * \left[\frac{i+2}{6}\right] + E_{SR}(N,L,P)$
DR	**L=2**	**L=3**
P=1	$\frac{1}{N} * \sum_{i=2}^{N/2} 2 * (\left[\frac{i-1}{4}\right] + 1)$	$\frac{1}{N} * (1 + \sum_{i=3}^{N/2} (\left[\frac{i-2}{6}\right] + \left[\frac{i}{6}\right] + 2))$
P=2	$\frac{1}{N} * (2 + \sum_{i=2}^{N/2} (6 * \left[\frac{i+3}{4}\right] - 2) + E_{DR}(N,L,P))$	$\frac{1}{N} * \sum_{i=1}^{N/2} (2 * \left[\frac{i}{2}\right] + 1) + E_{DR}(N,L,P))$

Table 1. $A_{T,N,L,P}$

SR	L=2	L=3
P=1	0	0
P=2	$2 * (\left[\frac{N-2}{4}\right] - \left[\frac{N-3}{4}\right])/N$	$(3 * (\left[\frac{N-3}{6}\right] - \left[\frac{N-4}{6}\right]) + (\left[\frac{N-2}{6}\right] - \left[\frac{N-3}{6}\right]) + (\left[\frac{N+2}{6}\right] - \left[\frac{N+1}{6}\right]))/N$
DR	**L=2**	**L=3**
P=1	0	0
P=2	$4 * (\left[\frac{N}{8}\right] - \left[\frac{N-1}{8}\right])/N$	$\frac{2}{N} * (\left[\frac{N+2}{12}\right] - \left[\frac{N+1}{12}\right]) + \frac{2}{N} * (\left[\frac{N-2}{12}\right] - \left[\frac{N-3}{12}\right])$

Table 2. $E_{T,N,L,P}$

	$m=1$	$m=2$	$m=3$
G_2	$G_1 + \left[\frac{N}{6}\right] + V_{DR,4}$	$G_1 + \left[\frac{N}{3}\right] + V_{DR,5}$	$G_1 + \left[\frac{2N}{3}\right] + V_{DR,6}$
G_3	$G_1 + G_2(m=1) + G_2(m=3)$	$G_1 + 2 * G_2(m=2) + V_{DR,7}$	$G_1 + G_2(m=1) + G_2(m=3)$

Table 3. Double Ring, Gateways Position When L=3

N	$V_{DR,4}$	$V_{DR,5}$	$V_{DR,6}$	$V_{DR,7}$
$6k$	0	0	0	0
$12k+2$	1	0	0	1
$12k+4$	1	0	0,1	0,1
$12k+8$	0	1	0,1	-1,0
$12k+10$	0	1	1	1

Table 7. $V_{DR,4}, V_{DR,5}, V_{DR,6}$ and $V_{DR,7}$ values

	SR
N	**Number of possibilities**
$2*L*k$	L*(L+1)+1
$L*(2*k+1)$	1
$Rest$	L

	DR ($1 \leq l_1 \leq L-1$)
N	**Number of possibilities**
$4*L*k$	2*(L-2)+1
$4*(L*k+l_1)$	2*L
$Rest$	L

Table 8. SR and DR Number of Optimal Configurations

of optimizing D_1, D_2, A_1 and A_2 in this order, Table 10 presents the solution.

Applying this method the three subnetworks will have almost the same performance parameters. Characterizing more regular topologies and implementing the formulas obtained, an algorithm can give, instantaneously, a solution for this kind of problems defining different goal functions . In this example the goal is to optimize the parameters, but other options could be to give a limitation on the diameters, the number of total gateways to form the higher level or many others.

6.2 Gateway Location

The optimal performance of a subnetwork is always related to the location of the gateways. The problem could seem trivial, the next example illustrates the opposite. The scenario consists on the previous result of the subnetwork with 12 nodes and this Single Ring is implemented optimizing the budget. Two gateways are required on this SR structure. This is the simplest problem of gateways location, chosen to demonstrate the application of this work even on simple designs. *A priori*, the best location for the gateways is as far as possible from each other. Hence, the number of nodes is divided by 2 and the result of the relative position of the gateways is G_1 and $G_2 = G_1 + 6$. The two nodes that are convenient to select as gateways (C_1 and C_2), for some different reasons such as largest cities, are relatively located from each other C_1 and $C_2 = C_1 + 5$.

Apparently, the two solutions are to discard one of the largest cities as gateway or redesign the ring so the two cities are relatively located in the proper way to be selected as gateways. The truth, based on the results at Table 4 and 6, is that there is no modification required since there are three different optimal relative gateway locations. G_1 and $G_2 = G_1 + 5$, $G_2 = G_1 + 6$ or $G_2 = G_1 + 7$. All this options obtain exactly the same values (optimal) for the diameters and average distances for the two independent paths, therefore they are considered optimal options.

The simplest example has shown that the gateway location is not trivial, hence, for more complex regular topologies this gateway analysis can be critical for the global network optimization, not only on the performance parameters but also in managing issues or economical investment.

N	12	20	30
$D_{SR,2,1}$	3	5	7
$D_{SR,3,1}$	2	3	5
$D_{DR,2,1}$	2	3	4
$D_{DR,3,1}$	2	2	3
$D_{SR,2,2}$	6	10	15
$D_{SR,3,2}$	4	7	10
$D_{DR,2,2}$	4	6	8
$D_{DR,3,2}$	3	4	6
$A_{SR,2,1}$	1.5	2.5	3.73
$A_{SR,3,1}$	1	1.65	2.5
$A_{DR,2,1}$	1.16	1.7	2.3
$A_{DR,3,1}$	0.83	1.25	1.7
$A_{SR,2,2}$	4.5	7.5	11.26
$A_{SR,3,2}$	3	5	7.5
$A_{DR,2,2}$	2.83	4.3	6.1
$A_{DR,3,2}$	2	3	4.2

Table 9. Scenario Parameters

N	$Topology$	D_1	D_2	A_1	A_2
30	DR L=3	3	6	1.7	4.2
20	DR L=2	3	6	1.7	4.3
12	SR L=2	3	6	1.5	4.5

Table 10. Solution

7 Conclusion

The multilevel study has returned some interesting conclusions. Before the parameters characterization, the procedure to plan the lowest level of a multilevel network was to test some potential options, calculate all the paths for all the possible transmissions and then decide the best option.

The studies of regular topologies as monolayer networks give some symmetries that can be useful for a multilevel scenario but always considering that the relations between the number of nodes and gateways will add complexity. Hence, the global network can be divided in subnetworks which are much easier to characterize.

The subnetwork of a multilevel structure can be characterized using regular topologies. The use of regular topologies allow to define parameters such as *average distance* and *diameter* as equations in function of the number of nodes *N* of the subnetwork, the gateways *L*, the path *P* and the *topology* considered. These parameters values follow well defined patterns and, therefore, they are deterministic in the way that the exact value of the diameter and the average distance can be estimated with no path calculation at all. The relative position of the gateways for obtaining the optimal performance of the subnetwork can be also characterized.

The properties of the potential network structures are obtained just using the newly given equations, or obtaining other equations for other topologies following the same methodology. These equations are useful to optimize the lowest level of the network in such way that each subnetwork has the same performance as the rest of the subnetworks. This balance on the performance allows for the optimization of the resources of the complete network and, hence, to take advantage of the network properties such as short diameters and short average distances and therefore short delays. The comparison of these options does not request a long or complex procedure, therefore, the planning can be focused on other important aspects such as the fiber civilian construction and implementation. The method is initially proved as possible with two topologies, Single and Double Ring, which motivates the research of the potential of the regular topologies for the multilevel issues.

The bases for an alternative methodology have been established. More topologies should be characterized as subnetworks and as the higher level, on a multilevel environment, to achieve a real feasible planning method.

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